AD-A246 164

TATION PAGE

Form Approved
OMB No 0704-0188

...to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, ...ewing the collection of information. Send comments regarding this burden estimate or any other aspect of this ...den to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson ...ce of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	January 22, 1991	Final Report 1/1/90 - 12/31/91

4. TITLE AND SUBTITLE

Nonlinear Waves and Inverse Scattering

5. FUNDING NUMBERS

N00014-90-J-1218

6. AUTHOR(S)

Mark J. Ablowitz

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

University of Colorado
Program in Applied Mathematics
Campus Box 526
Boulder, CO 80309-0526

8. PERFORMING ORGANIZATION REPORT NUMBER

153425

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Neil Gerr, Director of Math. Sciences
Department of the Navy
800 North Quincy Street
Arlington, VA 22217-5000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

N00014-90-J-1218

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Research in Nonlinear Waves and Inverse Scattering is continuing with many new and interesting results obtained. In this report a summary of the key areas of research is given and a list of research publications and preprints has been compiled. The main areas of interest include: multidimensional nonlinear wave equations of physical significance, multidimensional inverse scattering, numerically induced instabilities and chaos, and forced nonlinear wave problems.

92-02739

14. SUBJECT TERMS

Analysis of Nonlinear Equations, Inverse Scattering, Applications Thereof

15. NUMBER OF PAGES

7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
None	None	None	None

NSN 7540-01-280-5500

Standard Form 298 (Rev 2 89)
Prescribed by ANSI Std Z39-18
298-102

92 2 03 091

GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to *stay within the lines* to meet *optical scanning requirements*.

Block 1. Agency Use Only *(Leave blank)*.

Block 2. Report Date. Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least the year.

Block 3. Type of Report and Dates Covered. State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

Block 4. Title and Subtitle. A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

Block 5. Funding Numbers. To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

C	- Contract	**PR**	- Project
G	- Grant	**TA**	- Task
PE	- Program Element	**WU**	- Work Unit Accession No.

Block 6. Author(s). Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

Block 7. Performing Organization Name(s) and Address(es). Self-explanatory.

Block 8. Performing Organization Report Number. Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es). Self-explanatory.

Block 10. Sponsoring/Monitoring Agency Report Number. *(If known)*

Block 11. Supplementary Notes. Enter information not included elsewhere such as: Prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

Block 12a. Distribution/Availability Statement. Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NOFORN, REL, ITAR).

DOD -	See DoDD 5230.24, "Distribution Statements on Technical Documents."
DOE -	See authorities.
NASA -	See Handbook NHB 2200.2.
NTIS -	Leave blank.

Block 12b. Distribution Code.

DOD -	Leave blank.
DOE -	Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports.
NASA -	Leave blank.
NTIS -	Leave blank.

Block 13. Abstract. Include a brief *(Maximum 200 words)* factual summary of the most significant information contained in the report.

Block 14. Subject Terms. Keywords or phrases identifying major subjects in the report.

Block 15. Number of Pages. Enter the total number of pages.

Block 16. Price Code. Enter appropriate price code *(NTIS only)*.

Blocks 17. - 19. Security Classifications. Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

Block 20. Limitation of Abstract. This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

FINAL TECHNICAL REPORT

OFFICE OF NAVAL RESEARCH

NONLINEAR WAVES AND INVERSE SCATTERING

BY

MARK J. ABLOWITZ

GRANT #N00014-90-J-1218

JANUARY 29, 1992

Abstract

Research in Nonlinear Waves and Inverse Scattering is continuing with many new and interesting results obtained. In this report a summary of the key areas of research is given and a list of research publications and preprints has been compiled. The main areas of interest include: multidimensional nonlinear wave equations of physical significance, multidimensional inverse scattering, numerically induced instabilities and chaos, and forced nonlinear wave problems.

Summary of Research Activities

During the past 2 years we have had a very productive period in our research activities. There has been a total of 15 papers published, 8 preprints which either have been accepted or have been submitted for publication, and the PI has given 15 invited lectures all during the term of this research grant. In addition a monograph coauthored by the PI has just been published by Cambridge University Press summarizing the state of knowledge in this field today. A list of papers is attached to this report. The broad lines of our research investigations and brief discussion of the results are given below. Details may be found in the research papers.

Physically Significant Multidimensional Nonlinear Equations-Solutions and Inverse Scattering

We have been studying a class of multidimensional nonlinear evolution equations which can be solved by the method of the Inverse Scattering Transform (IST). Important equations include the Kadomtsev-Petviashvili (K-P), Davey-Stewartson (D-S), 2+1 Toda, and Self-Dual Yang-Mills (SDYM) equations. We have uncovered a number of interesting and important features associated with each of these equations. In the K-P equation we have established the particular form of nonlocal behavior that is needed in order to be able to solve the equation in infinite space with decaying initial data. For D-S we show how boundary data in a governing physical problem in which D-S is embedded as an asymptotic limit, fixes the correct boundary value problem for D-S. Both of the above equations: K-P and D-S are nonlocal and as such allow some freedom which requires proper analysis. For the 2+1 Toda system we have solved the appropriate initial value and boundary value problems by the DBAR method. The DBAR method was conceived as part of this grant eight years ago and has been employed by researchers world wide, ever since publication. The 2+1 Toda equation requires a combination of nonlocal Riemann-Hilbert and DBAR methods for its solution. The application of these ideas to solve boundary value problems is significant and new.

Since the 2+1 Toda system is discrete in one of its variables, we have consequently been able to apply the DBAR method to a partially discrete system. This paves the way for us to understand the inverse scattering analysis of other important discrete multidimensional systems.

Our studies of the SDYM system has demonstrated that reductions of SDYM result in well known soliton systems, classical integrable top-like systems as well as novel types of integrable systems such as the system of ODE's discovered by Halphen in the 1880's which may be transformed to an equation studied by Chazy (1910). The Chazy equation has a movable natural boundary in the complex plane and consequently extends the type of behavior usually found in integrable systems such as movable poles found in equations of the Painleve type. We have been able to find new partial differential equations which have reductions to the Halphen and Chazy equations and as such will have complicated behavior.

Numerically Induced Instabilities and Chaos

This avenue of research was begun by us in 1989 and has led to a number of important research results. We have shown that different numerical discretizations of various integrable equations, (e.g. the Nonlinear Schrodinger Equation) can lead to very different results depending on values of the mesh size. Depending on the numerical scheme employed, we have observed instabilities and chaotic structure. For sufficiently refined mesh size, all the schemes eventually converge and yield consistent results and the chaos and instabilities disappear. There are deep connections between these systems and the still burgeoning field of low dimensional chaos. Indeed these problems can be analyzed in great detail and yield insight as to how large degree of freedom systems can isolate particular degrees of freedom which generate the chaotic structure. Since the underlying equations and numerical schemes are of a Hamiltonian nature, it turns out that there are also important connections with symplectic numerical integrators which have been intensively studied during the past few years. We have been able to find the rate at which the chaotic structure "disappears" as the mesh is refined. Our symplectic integrators have chaotic structure which disappears exponentially fast as compared with the usual numerical schemes in which the chaos disappears at an algebraic rate-- of the order of the consistency of the numerical scheme.

Forced Nonlinear Wave Equations

In recent work we have studied nonlinear equations with forcing. The forcing typically takes the form of an external or parametric driver. In the externally forced problem we have analyzed the Burger's equation with delta function forcing and have shown how to solve the problem by both semi-infinite and infinite line methods. In the parametric forced problem we have investigated a Nonlinear Schrodinger type equation with time independent parametric forcing. It turns out that the bound states of the associated linearized Schrodinger system correspond to the travelling wave modes (solitons) which do not decay as time increases. The remaining part of the solution decays in time. We have recently analyzed a general physical system and have found a new Hamiltonian amplitude equation which governs modulated unstable wave systems. All of the above equations serve as prototypes of more general behavior in nonlinear wave equations and arise frequently in application.

BOOKS

Solitons, Nonlinear Evolution Equations and Inverse Scattering, M.J.Ablowitz and P.A. Clarkson, London Mathematical Society Lecture Notes Series #149, Cambridge University Press, Cambridge England, 1991.

PUBLICATIONS

1. On Homoclinic Structure and Numerically Induced Chaos for the Nonlinear Schrodinger Equation, M.J. Ablowitz and B.M. Herbst, SIAM J. Appl. Math., 50 (1990), 339-351.

2. Nonlinear Evolution Equations, Solitons, Chaos and Cellular Automata, M.J. Ablowitz, B.M. Herbst, J.M. Keiser, Research Reports in Physics, Nonlinear Physics, Ed. Gu, C., Li, Y. and Tu, G., Springer-Verlag, (1990), 166-189.

3. Nonlinear Evolution Equations, Inverse Scattering and Cellular Automata, M.J. Ablowitz,

Solitons in Physics, Mathematics and Nonlinear Optics, Springer-Verlag IMA Series, Vol. 25, (1990), p. 1-26.

4. Painleve Equations and the Inverse Scattering and Inverse Monodromy Transforms, M.J. Ablowitz, Solitons in Physics, Mathematics and Nonlinear Optics, Institute of Math. and its Applications, Springer-Verlag IMA Series, Vol. 25, (1990), p. 27-43.

5. One Dimensional Reductions of Self-Dual Yang-Mills Fields and Classical Equations, S. Chakravarty, M.J. Ablowitz, P.A. Clarkson, Phys. Rev. Lett. 65, (1990), 1085-1087.

6. Solitons, Numerical Chaos and Cellular Automata, M.J. Ablowitz, B.M. Herbst, J.M. Keiser, Integrable Systems, World Scientific, Ed. B. Kuperschmidt, (1990), p. 46-79.

7. On Homoclinic Boundaries in the Nonlinear Schrodinger Equation, M.J. Ablowitz and B.M. Herbst, Proc. of Workshop on Hamiltonian Systems, Transformation Groups, and Spectral Transform Methods, Les Publications CRM, Montreal, Canada, (1990), p. 121-132.

8. On the Boundary Conditions of the Davey-Stewartson Equation, M.J. Ablowitz, S.V. Manakov and C.L. Schultz, Phys. Lett. A, 148, (1990), 50-52.

9. On the Complete Integrability of Certain Nonlinear Evolution Equations in One and Two Spatial Dimensions, by M.J. Ablowitz and Javier Villarroel, Proc. Conference on Chaos and Order, Canberra, Australia,World Scientific, Ed. N. Joshi and R. Dewar (1991), Singapore.

10. Mel'nikov Analysis and Numerically Induced Chaos by B.M. Herbst and M.J. Ablowitz, PAM #19, to be published, conference on Chaos in Australia, Sydney, Australia (February 1990).

11. On the Kadomtsev Petviashvili Equation and Associated Constraints, M.J. Ablowitz and Javier Villaroel, Studies in Appl. Math. 85, (1991), 195-213.

12. On the Hamiltonian Formalism for the Davey-Stewartson System, Javier Villaroel and M.J. Ablowitz, Inverse Problems 7, (1991), 451-460.

13. One Dimensional Reductions of Self-Dual Yang-Mills Fields and Classical Equations, S. Chakravarty, M.J. Ablowitz and P.A. Clarkson, published in Einstein's Studies Series, Vol. 4, "Recent Advances in Relativity-- Essays in Honor of Ted Newman", Ed. Allan I. Janis and John R. Porter, Birkhauser Press, 1992, 60-71.

14. On Reductions of Self-Dual Yang Mills Equations, S. Chakravarty and M.J. Ablowitz, Proceedings on Painleve Equations and their Applications, NATO Conference, 1990, PAM #62, (September 1990).

15. Numerical Homoclinic Instabilities and the Complex Modified Korteweg-De Vries Equation, B.M. Herbst, M.J. Ablowitz and E. Ryan, Computer Phys. Commun. 65, 1991, 137-142.

16. Forced and Semiline Solutions of Burger's Equation, by M.J. Ablowitz and S. de Lillo, Phys. Lett. A, 156, 1991, 483-487.

17. Nonlinear Wave Propagation, Encyclopedia of Physics, 2nd Edition, 1991, Ed. R.G. Lerner, G.L. Trigg, VCH Publishers, Inc., New York.

PREPRINTS

1. On Particles and Interaction Properties of the Parity Rule Filter Automata, by M.J. Ablowitz and J.M. Keiser, PAM #68, (January 1990).

2. Numerical Chaos, Symplectic Integrators and Exponentially Small Splitting Distances, by B.M. Herbst and Mark J. Ablowitz, PAM #78, (June 1991), submitted to J. Comp. Phys.

3. Exponentially Small Splitting Distances in Leapfrog Discretizations of Nonlinear Dynamical Systems, by B.M. Herbst, Mark J. Ablowitz and J.M. Keiser, PAM #105, (September 1991).

4. On the Method of Solution to the 2+1 Toda Equation, J. Villaroel and M.J. Ablowitz, PAM #107, (October 1991), accepted Phys. Lett. A.

8. Self-Dual Yang-Mills Equations and New Special Functions in Integrable Systems, S. Chakravarty, M.J. Ablowitz and L.A. Takhtajan, PAM #108 (October 1991) to be publ. proceedings NEEDS Conference, Gallipoli, Italy, July 19-29, 1991.

5. Solitons and Computation, M.J. Ablowitz and B.M. Herbst, PAM #110 (November 1991), accepted Springer-Verlag collection on Advances in Soliton Theory by A.S. Fokas and V.B. Zakharov.

7. Integrable Systems, Self-Dual Yang-Mills Equations and Connections with Modular Forms, M.J. Ablowitz, S. Chakravarty and L. Takhtajan, PAM #113, proceedings Symposium on Nonlinear Problems in Engineering and Science, Tsinghua University, Beijing, China, October 16-20, 1991.

6. A New Hamiltonian Amplitude Equation Governing Modulated Wave Instabilities, M. Wadati, H. Segur and M.J. Ablowitz, PAM #114 (December 1991), submitted J. Phys. Soc. Japan.

PAM: Program in Applied Mathematics Preprint

INVITED LECTURES

1. Oberwolfach, W. Germany, Jan. 13-20, 1990, "Aspects of Integrability and Chaos".

2. Workshop on Chaos and Order, Australian National University, Canberra, Australia, Feb. 1-3, 1990, "Integrability and Cellular Automata".

3. Conference on Chaos, University of South Wales, Sydney, Australia, Feb. 4-10, 1990, "Numerically Induced Chaos".

4. University of Colorado at Denver, Mathematics Dept., Feb. 28, 1990, "Solitons and All That Nonlinear Stuff".

5. Conference on Recent Advances in General Relativity, University of Pittsburgh, May 3--5, 1990, "Integrability, Reductions of Self-Dual Yang Mills Fields and Classical Systems".

6. University of Georgia, Physics Department, Athens, GA, May 6--9, 1990, "Integrability, Chaos and Patterns".

7. SIAM National Conference, Minisymposium on Inverse Scattering, "Multidimensional and Nonlinear Evolution Equations", Chicago, Illinois, July 15--17, 1990.

8. NATO Advanced Research Workshop, Sept. 3-7, 1990, Montreal, Canada, "One Dimensional Reductions of Self-Dual Equations".

9. Aspects of Nonlinear Dynamics: Solitons and Chaos; Brussels, Belgium, Dec. 6-8, 1990, "Solitons, Homoclinic Orbits and Numerically Induced Chaos".

10. Southern Methodist University, Department of Mathematics, Feb. 23-25, 1991, "Solitons and all that Nonlinear Stuff".

11. Workshop on Nonlinear Evolution Equations and Dynamical Systems, June 24-29, 1991, Gallipoli, Italy, "Reductions of the Self Dual Yang-Mills equations and Modular Forms."

13. NATO Advanced Scientific Workshop, July 9-18, 1991, Patras, Greece, "Solitons, Computation and Chaos."

12. Symposium on Nonlinear Problems in Science and Engineering, Tsinghua University, Beijing, China, Oct. 16-20, 1991, "Self-Dual Yang-Mills, Solitons and Connections with Modular Forms".

13. Kao Corporation, Tokyo Japan, Oct. 22, 1991, "Solitons, Chaos and Cellular Automata".

14. University of Tokyo, Japan, Physics Department, "Reductions of Self-Dual Yang-Mills Equations and Connections with Modular Forms", Oct. 23, 1991.

15. Ryukoku University, Seta, Japan, Applied Mathematics Dept., "Numerical Chaos and Cellular Automata", Oct. 28, 1991